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Verification of closed-form solutions of fatigue life under along-wind loading

A.P. Robertson [a,*], J.D. Holmes [b,c], B.W. Smith [b]

[a] *Silsoe Research Institute, Wrest Park, Silsoe, Bedford MK45 4HS, UK*
[b] *JDH Consulting, P.O. Box 269, Mentone, Vic., Australia*
[c] *Flint and Neill Partnership, 53 Great Suffolk Street, London SE1 ODB, UK*

Abstract

Records of wind-induced strains measured in 12 m high, tubular steel lighting columns have been collected over a range of mean wind speeds up to 17 m/s. Since this did not embrace the full transcritical range of Reynolds numbers, a novel approach was adopted to acquire a more complete data set. The diameter of one of the columns was increased from 0.14 to 0.58 m by means of plastic drums mounted so as not to influence the stiffness or strength of the column. The back-corrected data harmonised with the original data set. The full data set has been used to verify recently-proposed, simple, closed-form solutions of fatigue damage and fatigue life in structures subjected to along-wind loading. The closed-form model offers upper and lower-bound solutions corresponding to narrow-band resonant vibrations and wide-band background response, respectively. The solutions provide remarkably good predictions of the experimentally derived damage rates across the full range of Reynolds numbers tested, particularly when the shallower part of the bi-linear fatigue endurance curve for the shoulder-weld of concern is adopted in the model. Predictions of fatigue life are similarly promising in relation to the actual performance of similar lighting columns at sites with different wind exposures.
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1. Introduction

Turbulent winds produce fluctuating loads on structures that can cause fatigue problems. Lighting columns are an example of a structure for which along-wind buffeting loading (exacerbated by Reynolds drag instability) has produced a number of incidents of premature fatigue failure at exposed sites in the UK (Fig. 1). These failures prompted research to be undertaken at full-scale to measure the cycles of stress in 12 m high, tubular steel lighting columns with respect to approach wind speed [1], in order to appraise and develop design rules. Independently, Holmes [2,3], developed simple, closed-form solutions for the prediction of fatigue damage to, and fatigue life of structures subjected to along-wind loading that take into account both the resonant and background responses. Such closed-form solutions are potentially hugely attractive and valuable to engineers, but the solutions need to be validated through calibrations with real data. The full-scale testing of lighting columns at Silsoe Research Institute (SRI) generated eminently suitable data sets with which to calibrate the recently developed solutions. This paper compares results obtained from the solutions with those obtained from the full-scale measurements.

2. Full-scale measurements of wind-induced fatigue loading and damage rates

2.1. Phase I measurements

Phase I of the full-scale measurements on lighting columns have been reported [1]. The tubular steel lighting columns erected at the SRI Wind Engineering site (Fig. 2) were 12 m in height and each supported at

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* Corresponding author. Tel.: +44-1525-860000; fax: +44-1525-861735.

  E-mail address: adam.robertson@bbsrc.ac.uk (A.P. Robertson).

Fig. 1.   Fatigue crack in a lighting column in service.

Fig. 3.   Fatigue endurance (s–N) curve for shoulder weld of lighting column.

its top a single 7.8 kg luminaire mounted on a short arm such that the outstand of the centre of the luminaire was 0.8 m from the column. Each column was strain-gauged just above the shoulder-weld approximately 1.6 m above the ground where the 0.19 m diameter base tube reduced to the 0.14 m in diameter by 3 mm thick main shaft tube. Simultaneous 30-min records of bending strain in two planes, and of wind speed at 10 m height were collected for ranges of wind speed and direction.

The strain records were converted to stress and cycle-counted using 'rainflow' analysis into 5 MPa bins. These stress cycles were translated into damage using a fatigue endurance (s–N) curve (Fig. 3) for the

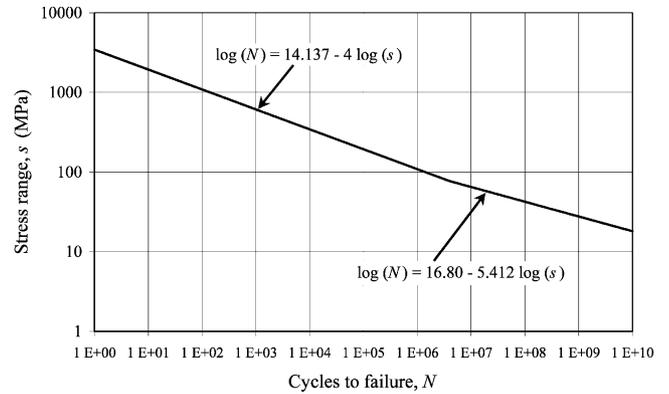Fig. 2.   Experimental lighting columns at the SRI test site.

shoulder weld. A damage correction for mean stress was applied (Goodman mean stress correction) but this was found to have an insignificant effect on the results.

The fatigue endurance curve was obtained from a series of sinusoidal loading tests on representative lighting columns conducted at TWI (formerly The Welding Institute) in the UK [4]. The loading cycles were of fixed amplitudes with maximum/minimum stress ratios of −1, and so had a mean stress of zero. The stress amplitudes were set to give an expected spread of lives in the range of 10^5 to 2×10^6 cycles. The cycling rate varied from 1 to 6 Hz depending on the stiffness of the column tested. The results from this exercise have been incorporated into the UK and European Standards for lighting column design (BD26/99 [5] and prEN 40 [6]).

The data records enabled damage/hour to be plotted with respect to mean wind speed. The records were binned according to the mean wind speed into bins of 2 m/s width. The damage results were averaged for each bin and plotted with respect to the central value for each mean wind speed bin. The resulting damage data from Phase I of the measurements are shown in Fig. 4. These results are for two similar columns, one planted in the ground and the other bolted through a flange plate to a substantial concrete foundation. The two sets of results were very similar to each other. The strain records in Phase I were sampled at 12 Hz (on selected runs, data were sampled instead at 20, 40 or 585 Hz). Subsequent measurements (Phase II) where the sampling rate was increased to 100 Hz to enhance the capture of cycle peaks, and the sampling time was reduced to 10 min, confirmed the original Phase I measurements. The highest mean wind speeds for which data were obtained fell in the 17 m/s bin. This was not a sufficiently high wind speed to embrace the full transcritical range of Reynolds number loading effects, but higher wind speeds were unlikely to arise within an acceptable timescale. Consideration was given to how
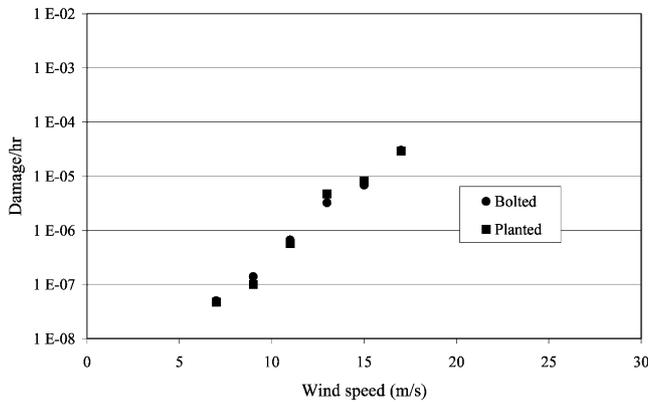
Fig. 4. Damage data for bolted-base and planted lighting columns.

Fig. 5. Plastic drums used to enlarge the diameter of one of the lighting columns.

data for higher wind speeds might be obtained. Monitoring a different column on a more exposed site in the north-west of the country was a possibility but this was discounted because of the cost, security risk, and the low probability that even on another site significantly higher (i.e. approximately 50% higher) wind speeds would arise within a reasonable timescale. Instead, a more novel approach was taken for Phase II of the measurements.

2.2. Phase II measurements

It was decided to enlarge the diameter of a second, planted column and to retain the existing planted column as a reference. It was important to enlarge the diameter in a way that did not stiffen or strengthen the column. This was achieved by using 12 polypropylene drums, each 0.93 m height and 0.58 m in diameter. Holes of 0.14 m diameter were cut centrally in the top and bottom flanges of the drums, and the luminaire and its arm were removed from the top of the column so that the drums could be fed on to the shaft (Fig. 5). Small spacer rings were inserted between adjacent drums so that the flexural stiffness of the column would not be influenced by the presence of the drums. Thus the diameter of the column was increased from 0.14 to 0.58 m (i.e. by a factor of 4.1 on diameter and hence on face area). This reduced the natural frequency of the column from 0.89 to 0.66 Hz in the plane of luminaire, and to 0.70 Hz in the orthogonal plane. The damping was also found to have reduced slightly from a typical log decrement value of 0.06 for the virgin column to 0.04. Adding the drums also changed the vortex-shedding frequency; and the wind speed at which the vortex-shedding frequency coincided with the natural frequency of the column changed from approximately 0.6 to 1.9 m/s, both being well below the wind speeds of interest in this study.

Measurements were repeated, this time at a sampling rate of 100 Hz and for a 10 min record length. Fatigue data were obtained for different wind directions and for mean wind speeds of up to 18 m/s. The data were processed to obtain damage data as before. However, these data need to be back-corrected in order for them to apply to the original 0.14 m diameter column.

Principally, the data need to be corrected in relation to the wind speed at which the damage levels arise, but the damage magnitudes also need to be corrected in relation to the change in natural frequency, which resulted from the additional mass of the drums. The reduction in natural frequency produced by adding the drums means that fewer cycles are undergone in a given time for a given wind speed. The damage magnitudes were thus amplified by the ratio of the change in natural frequency, i.e. by the factor 1.3 (0.89/0.68). The correction for wind speed in the absence of a Reynolds number effect would simply be the square-root of the area ratio, i.e. $\sqrt{4.1} = 2.02$. If the Phase II damage data corrected in this way are plotted on the same axes as used in Fig. 4, the plot shown in Fig. 6 is obtained. Here the data from the original planted column have been transposed from Fig. 4, so all the data shown in Fig. 6 relate to a planted column of the same design. The discontinuity or disparity between the two data sets is due to the drag coefficient dependence on Reynolds number effects arising from the change in diameter and surface roughness of the cylinder.

A Reynolds number correction to the data set was pursued through reference to ESDU 80025 [7], from where the curves shown in Fig. 7 were derived. The
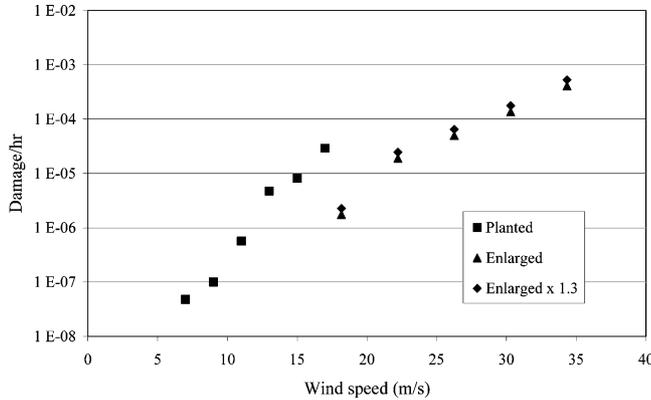
Fig. 6. Damage data for original planted column and for enlarged-diameter planted column with and without augmentation for natural frequency change and when wind speeds have been corrected for face area only.

Table 1
Damage/hour results obtained from enlarged-diameter column

Wind speed band central value, V_E (m/s)	Damage/hour	Wind speed corrected for area and Re effect, V_O (m/s)	Damage/hour corrected for natural frequency
9	1.75×10^{-6}	13.3	2.27×10^{-6}
11	1.90×10^{-5}	16.7	2.46×10^{-5}
13	4.97×10^{-5}	19.2	6.47×10^{-5}
15	1.36×10^{-4}	21.9	1.77×10^{-4}
17	4.06×10^{-4}	24.5	5.28×10^{-4}

upper two curves relate to the original, 0.14 m diameter galvanised steel column, the uppermost curve being for an assumed surface roughness, ε, of 1×10^{-4} m, and the lower of the two being for an assumed surface roughness, ε, of 0.8×10^{-4} m. The lowest curve in Fig. 7 relates to the enlarged, 0.58 m diameter column for which a smoother surface with a roughness parameter of 3×10^{-6} m was assumed for the plastic drums. The x-axis of Fig. 7 shows Reynolds number, Re, and the wind speeds, V, for the two different diameter, D, columns that equate to the Re values. A given wind speed, V_E, experienced by the enlarged column corresponds to a drag coefficient, C_{DE}, given by the bottom line in Fig. 7. In the absence of a Reynolds number effect, the wind speed V_E experienced by the enlarged column would equate to a wind speed of 2.02 V_E experienced by the original column. At a wind

speed of 2.02 V_E, the drag coefficient applicable to the original column increases to C_{DO}, taken here as the value midway between the upper two curves in Fig. 7. Since the original column would experience a higher drag loading, the damage measured on the enlarged column at a wind speed of V_E will be an underestimate of the damage experienced by the original column at a wind speed of 2.02 V_E. Consequently, this damage level equates to that experienced by the original column at a lesser wind speed, V_O. To a first approximation, this wind speed (V_O) is given by factoring the area-corrected wind speed of 2.02 V_E by the drag coefficient ratio C_{DE}/C_{DO}, i.e.

$$V_O = 2.02 \frac{C_{DE}}{C_{DO}} V_E$$

The C_{DE}/C_{DO} ratios corresponding to the five binned wind speed values for which damage data were collected on the enlarged column vary between 0.72 and 0.76. The original and corrected wind speeds and damage values are given in Table 1. When the fully corrected data are plotted, the data are found to collapse very well onto the trend of the original data set from the original column, as shown in Fig. 8.
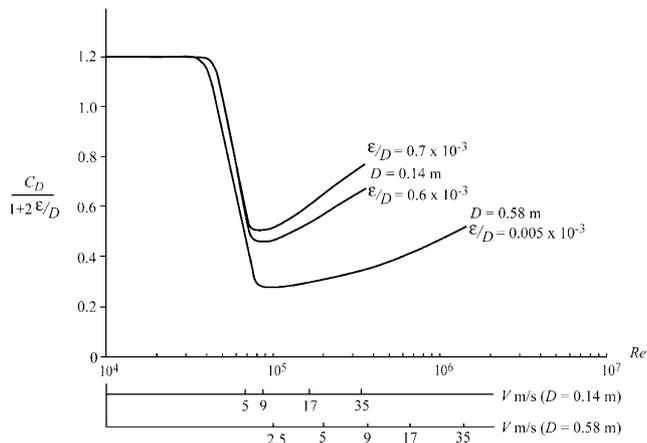

Fig. 7. Drag coefficients C_D for original 0.14 m diameter steel column and enlarged 0.58 m plastic-drum column as a function of Reynolds number Re where ε is the surface roughness of the cylinders (from ESDU 80025).
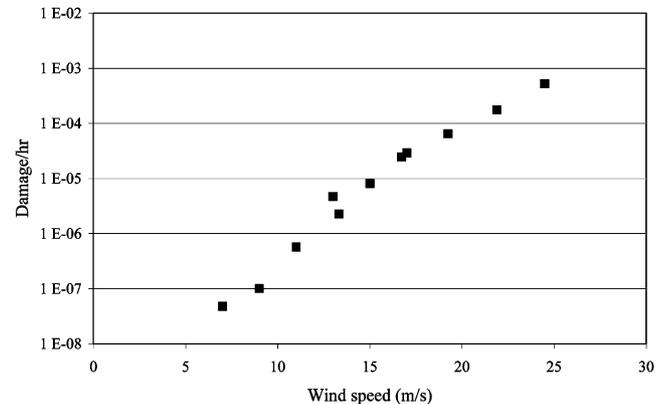

Fig. 8. Damage data for original planted column and for enlarged-diameter column when corrected for natural frequency, face area and Reynolds number effect.

3. Closed-form solutions for fatigue damage

Holmes [2,3] has presented a simple closed-form solution for predicting fatigue damage to, and fatigue life of, structures subjected to along-wind loading. Miner's Rule is the criterion adopted to define failure under cyclical loading of different amplitudes. Expressions have been derived for the narrow-band, resonant response, and for the wide-band, background response. The expression for fatigue damage rate (slightly modified in terms of stress range rather than amplitude) is:

$$\frac{D_a}{T} = \frac{\lambda v_o^+}{K}\left(2\sqrt{2}\sigma\right)^m \Gamma\left(\frac{m}{2}+1\right) \tag{1}$$

where λ is a factor equal to 1.0 for the narrow-band, upper-bound value, and less than 1 for the wide-band, lower-bound value, and v_o^+ is the cycling rate (taken as the natural frequency of the structure, 0.89 Hz, for the upper bound, and half the natural frequency for the lower bound). The parameters K and m are obtained from the equation for the s–N curve (s being stress range and N the number of cycles). The s–N curve obtained for the shoulder weld of representative lighting columns from the sinusoidal tests was bi-linear (Fig. 3). Taking the left-hand (steeper) part of the curve gives:

$$K = Ns^m = Ns^4 = 10^{14.137} \text{ (MPa)}^4$$

Γ is the gamma function, and σ is the standard deviation of stress which is a function of the mean wind speed $\bar{U}$, given by:

$$\sigma = A\bar{U}^n \tag{2}$$

The constants A and n in Eq. (2) were found by evaluating the standard deviations of selected strain records and plotting these against the corresponding mean wind speeds, which gave $A = 0.152$ and $n = 1.55$. For the lower bound estimation, $\lambda = 0.926$–0.033 m [3]. Taking logarithms of Eq. (1) and substituting the above values gives the damage rates (per second) for the lighting column as:

$$\begin{aligned}\log_e \frac{D_a}{T} &= -35.35 + 6.2\log_e\bar{U} \text{ (Upper limit)} \\ \log_e \frac{D_a}{T} &= -36.27 + 6.2\log_e\bar{U} \text{ (Lower limit)}\end{aligned} \tag{3}$$

4. Comparisons of experimental and theoretical damage rates

Fig. 9 shows the upper and lower predictions obtained from Eq. (3) when the left-hand, steeper, part of the s–N curve (Fig. 3) is taken to apply across the full range of cycles in the evaluation of K and m in Eq. (1). The predictions represent reasonably good bounds to the experimental fatigue damage results over
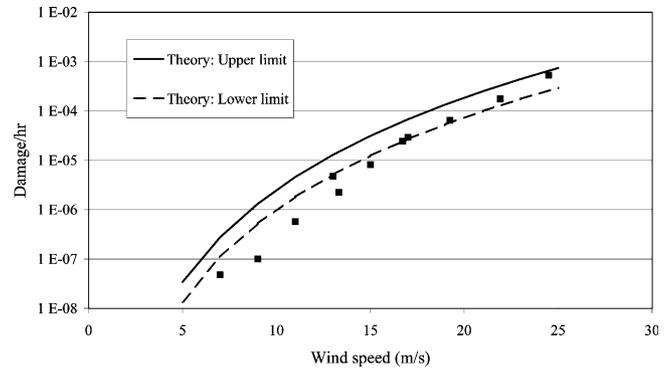

Fig. 9. Comparison of full-scale, experimentally derived fatigue damage rates and closed-form predictions using the steeper part of the s–N curve (Fig. 3).

the higher wind speeds. At the lower wind speeds, the theoretical bounds are high, which is unsurprising considering the steeper part of the s–N curve was used in the calculation.

If, instead, the right-hand, shallower, part of the s–N curve (Fig. 3) is taken to apply over the full range of cycles in the evaluation of K and m, the predictions shown in Fig. 10 are obtained.

These predictions are exceedingly good, and this is presumably associated with the majority of the damage encountered in the experimental work being attributable to cycles of small stress range represented by the shallower part of the s–N curve. Unfortunately, a bi-linear s–N curve such as that in Fig. 3 introduces a discontinuity into the integral underlying the derivation of Eq. (1) that renders the integral too complex to solve whilst retaining a simple closed-form solution.

In practical design situations where a bi-linear s–N curve applies, the steeper part of the curve should be used for estimates of the upper limit of fatigue damage rate and lower limit of design life (see the following section) as this will be more conservative.

If the exponent n in Eq. (2) is assigned its theoretically expected minimum value of 2 [3], rather than the
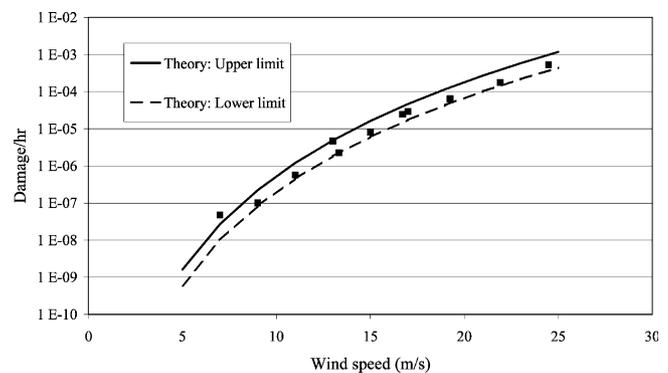

Fig. 10. Comparison of full-scale, experimentally derived fatigue damage rates and closed-form predictions using the shallower part of the s–N curve (Fig. 3).

Fig. 11. Comparison of full-scale, experimentally derived fatigue damage rates and closed-form predictions using the shallower part of the s–N curve (Fig. 3) and an exponent n value in Eq. (2) of 2 instead of 1.55.
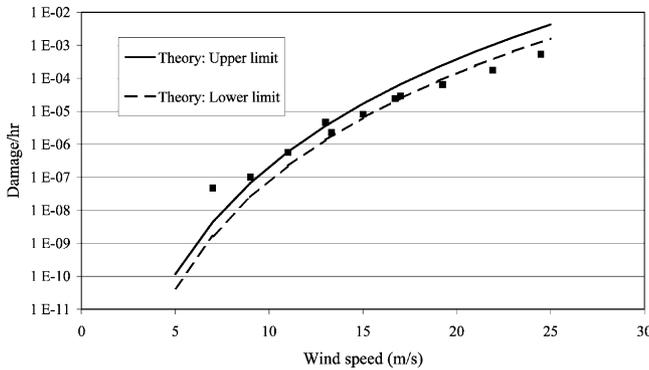
experimentally derived value of 1.55, and the prediction is re-calculated using the shallower part of the s–N curve, the agreement is not as good, as shown in Fig. 11. The reason for the lower experimental exponent value may be the 'cross-talk' phenomenon exhibited by the column [1] whereby oscillations excited by loading in one plane invoked oscillations in the orthogonal plane, which then fed back to oscillations in the original plane again etc. This 'switching' or 'cross-talk' arose because of an access hole present in the base of the column that was responsible for two very slightly different fundamental frequencies existing in the two planes, and because of torsional effects introduced by the luminaire outstand.

5. Predicted fatigue lives

When combined with a Weibull distribution representing the population of wind speeds to which a column is likely to be exposed, the experimental fatigue damage results can be used to obtain a prediction of fatigue life.

Analytically, Holmes [3] gave expressions for upper and lower limits to fatigue life. The following are slightly adjusted forms of those in [3] to allow for the s–N parameters being expressed in terms of stress range rather than amplitude:

$$T_{\mathrm{lower}} = \frac{K}{v_{\mathrm{o}}^{+}\left(2\sqrt{2}A\right)^{m}c^{mn}\Gamma(m/2+1)\Gamma((mn+k)/k)} \quad (4)$$

$$T_{\mathrm{upper}} = \frac{K}{\lambda v_{\mathrm{o}}^{+}\left(2\sqrt{2}A\right)^{m}c^{mn}\Gamma(m/2+1)\Gamma((mn+k)/k)} \quad (5)$$

where c and k are parameters of the Weibull probability distribution fitted to the reference mean wind

speed at the site in question, in the form:

$$f_{\mathrm{U}}(\bar{U}) = \frac{k\bar{U}^{k-1}}{c^{k}}\exp\left[-\left(\frac{\bar{U}}{c}\right)^{k}\right] \quad (6)$$

For sites in the UK, Robertson et al. [1] assumed a value for k of 1.85, and a relationship for c in Eq. (6) with the 50-year return-period hourly wind speed, $\bar{U}_{50}$ of:

$$c = \bar{U}_{50}(12.99)^{-1/k} \quad (7)$$

According to the British Standard for towers and masts [8], $\bar{U}_{50}$ is 24.9 m/s for the test site at Silsoe, and Eq. (7) gives c equal to 6.23 m/s. For exposed sites in the north of England, where premature fatigue of lighting columns has been experienced, $\bar{U}_{50}$ is about 32.5 m/s, and c is then 8.13 m/s. Substitution into Eqs. (4) and (5), using the parameters from the steeper part of the fatigue endurance curve (Fig. 3), then gives the upper and lower estimates of fatigue life listed in Table 2.

The estimated life of 1–3 years at exposed sites in the north of England corresponds reasonably with the experience of certain lighting columns located at exposed sites in that region, although these were considered to be distinctly premature failures. It should be borne in mind, however, that stronger columns than those installed at Silsoe may be necessary in the north of England to withstand the correspondingly higher 'quasi-static' wind loading requirements. A greater life is predicted for the same columns at the less exposed Silsoe site in south-eastern England. These estimates are also on the low side compared with experience. These predictions are obtained using the steeper part of the s–N curve in Fig. 3 which means that the damage at higher cycles associated with lower wind speeds is overestimated, and so the fatigue life is underestimated. If instead the shallower part of the s–N curve is used, the damage at lower cycles associated with higher wind speeds is overestimated, and so the fatigue life is again underestimated, but more so. If it were possible to derive a solution that used both parts of the endurance curve, higher fatigue life estimates would result. It is likely that the endurance curve provides a conservative representation of fatigue damage in relation to most practical columns, and this too would lead to under-

Table 2
Estimates of fatigue life

Site	$\bar{U}_{50}$ (m/s)	c (m/s)	T_{lower} (years)	T_{upper} (years)
Northern England, exposed	32.5	8.13	1.1	2.7
Silsoe, SE England, less exposed	24.9	6.23	5.6	14.1

estimations of actual fatigue lives. Additionally, it is possible that the experimental records included in this study did not, with appropriate weighting, represent fully the wind directionality effects for all wind speeds.

6. Conclusions

Experimental measurements of wind-induced fatigue loading on 12 m high tubular steel lighting columns under natural winds have been completed. The maximum mean wind speeds during the original measurement period was 17 m/s which limited the range of Reynolds number for which data were obtained. The novel approach of enlarging the column diameter with discrete plastic drums to obtain data over a fuller range of Reynolds numbers appears to have been successful. Conversion of the strain time-histories obtained into damage rates using rainflow cycle-counting and an experimentally derived fatigue endurance curve has enabled the relationship between fatigue damage and wind speed to be determined.

These data have been used to verify a simple theoretical model of fatigue damage rate and fatigue life. Theoretical damage rates expressed as a function of mean wind speed agree remarkably well with rates derived directly from the experimental data. Theoretical fatigue-life estimates appear to be reasonable, if a little conservative, in relation to experience of lighting columns in service in England. The conservatism lies to some extent in the limitation that only one of the two parts of the bi-linear endurance curve can be embraced in the closed-form model, but may also arise from any conservatism inherent within the fatigue endurance curve itself.

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