SHORT COMMUNICATION

Mode shape corrections for dynamic response to wind

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A correction factor, for nonlinear mode shapes, to the generalized force spectra in the first mode of vibration produced by wind action on tall structures, is proposed. This lies between theoretical limits for low and high correlation of the wind forces with height and agrees well with experimental data.

It is now standard practice to carry out wind tunnel tests of tall buildings greater than 30–40 storeys in height for the dynamic response induced by wind action. Such tests are either aeroelastic tests in which the inertia and stiffness properties of the structure are modelled[1] or force balance tests designed to determine the generalized force spectrum in the first mode of vibration.[2] Both the force balance tests and aeroelastic tests in which a rigid model, pivotted at the base, is used, produce information relevant to a building with a fundamental mode shape varying linearly with height in the form:

$$\Psi(z) = \frac{z}{h} \tag{1}$$

Since many tall buildings have mode shapes with significant nonlinearity, it is desirable to have a method of correcting the information for nonlinear mode shapes.

Several codes of practice for wind loading now include a method for taking account of along-wind dynamic response of tall structures.[3,4] There are also proposals to include cross-wind dynamic analysis in future codes.[5] Such methods are particularly useful for preliminary design before expensive wind tunnel tests are undertaken. Again, these methods are invariably based on data appropriate to a linear mode shape, and mode shape corrections are useful for these methods to broaden their applicability.

Mode shape corrections for the dynamic response of tall buildings have previously been considered by Saunders and Melbourne[6], Kwok[7], Kareem[8], and Vickery *et al.*[9]. It is the purpose of this note to consider, more fully, theoretical limits to the mode shape correction factor and to suggest a simple formula which is suitable for use in a code. As in the previous studies, a power

law with exponent, β, will be taken as the general form for mode shape, i.e.:

$$\Psi(z) = \left(\frac{z}{h}\right)^{\beta} \tag{2}$$

Low correlation limit

The mode shape correction factor will be taken as a correction to the generalized force spectrum in the first mode of vibration, which can be written as:

$$S_G(n) = \int_0^h \int_0^h Co(z_1, z_2, n)\Psi(z_1)\Psi(z_2)\, dz_1\, dz_2 \tag{3}$$

with

$S_G(n)$	generalized force spectrum
$Co(z, z_2, n)$	co-spectrum of fluctuating wind forces
n	frequency
z_1, z_2	separate height coordinates
h	height of building

It should be noted that this form is applicable to both along-wind and cross-wind dynamic forces. Clearly, from equation (3), the general equation for the correction factor, F, to correct the generalized force for a linear mode shape to that for an arbitrary mode shape is:

$$F = \int_0^h \int_0^h Co(z_1, z_2, n)\Psi(z_1)\Psi(z_2)\, dz_1\, dz_2 \Bigg/ $$
$$\int_0^h \int_0^h Co(z_1, z_2, n)\left(\frac{z_1}{h}\right)\left(\frac{z_2}{h}\right)\, dz_1\, dz_2 \tag{4}$$

The value of F will thus depend on the form of the co-spectrum $Co(z_1, z_2, n)$ as well as the mode shape $\Psi(z)$.

Consider the following form for the spectrum:

$$Co(z_1, z_2, n)$$
$$= (S_f(z_1, n))^{1/2}(S_f(z_2, n))^{1/2}R(z_1 - z_2, n)$$

where $S_f(z, n)$ is the spectrum of the force per unit height and $R(z_1 - z_2, n)$ is a cross-correlation function which depends on the separation distance, $z_1 - z_2$.

Now, changing the variable z_2 in the double integral on the right-hand side of equation (3) to $\xi = z_1 - z_2$, the generalized force spectrum becomes:

$$S_G(n) = \int_0^h \int_{-z_1}^{h-z_1} (S_f(z_1, n))^{1/2}(S_f(z_1 - \xi, n))^{1/2}$$
$$\times R(\xi, n)\Psi(z_1)\Psi(z_1 - \xi)\,dz_1\,d\xi \quad (5)$$

Now, consider the limiting case where the correlation falls off rapidly with ξ, i.e. the correlation length is very small in relation to the height h. Then, equation (5) can be written as:

$$S_G(n) \simeq \int_0^h S_f(z_1, n)\Psi^2(z_1)\,dz_1 \int_{-\infty}^\infty R(\xi, n)\,d\xi$$

$$= \lambda(n) \int_0^h S_f(z_1, n)\Psi^2(z_1)\,dz_1 \quad (6)$$

where $\lambda(n) = \int_{-\infty}^{+\infty} R(\xi, n)\,d\xi$, the correlation length.

Then, substituting for low correlation in equation (4):

$$F_1 = \frac{\lambda(n) \int_0^h S_f(z_1, n)\Psi^2(z_1)\,dz_1}{\lambda(n) \int_0^h S_f(z_1, n)\left(\frac{z_1}{h}\right)^2\,dz_1}$$

$$\simeq \frac{\int_0^h \Psi^2(z)\,dz}{\int_0^h \left(\frac{z}{h}\right)^2\,dz} \quad (7)$$

provided that the spectrum of sectional forces $S_f(z, n)$ does not vary greatly with z.

High correlation limit

A second limit can be obtained by setting:

$$Co(z_1, z_2, n) = \text{constant}$$

in equation (4).
Then:

$$F_2 = \int_0^h \Psi(z_1)\,dz_1 \int_0^h \Psi(z_2)\,dz_2 \Bigg/$$

$$\int_0^h \left(\frac{z_1}{h}\right)\,dz_1 \int_0^h \left(\frac{z_2}{h}\right)\,dz_2$$

$$= \left[\int_0^h \Psi(z)\,dz\right]^2 \Bigg/ \left[\int_0^h \left(\frac{z}{h}\right)\,dz\right]^2 \quad (8)$$

Note that this assumption implies not only full correlation of the dynamic forces over the height of the structure, but also a uniform sectional force spectrum with height.
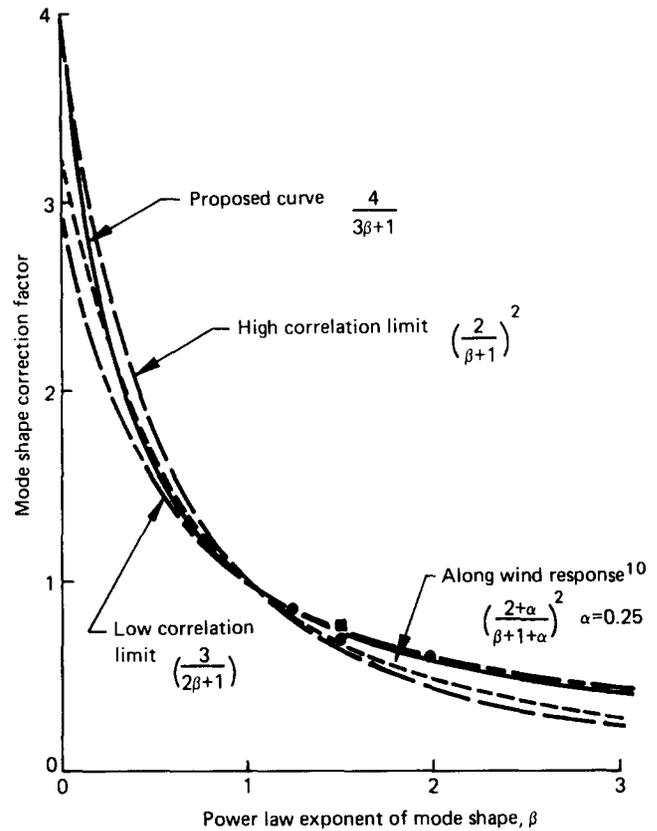

Figure 1 Mode shape correction factor: (●) UWO urban; (■) UWO open country

Proposed correction factor

The functions F_1 and F_2 can be evaluated for the power law mode shape of equation (2) as follows. For low correlation:

$$F_1 = \frac{3}{2\beta + 1} \quad (9)$$

For high correlation:

$$F_2 = \left(\frac{2}{\beta + 1}\right)^2 \quad (10)$$

These functions are plotted in *Figure 1* against the exponent, β. It can be seen that these functions are relatively close to each other, despite the differences in the assumptions in the co-spectrum. The curves cross at a value of unity for β equal to unity.

The following is proposed as an intermediate function suitable for use in a code:

$$F_0 = \frac{4}{3\beta + 1} \quad (11)$$

As well as being very simple, this function lies between the two limiting functions for all β, as shown in *Figure 1*, but tends towards the conservative limit, which is desirable for code usage. F_0 also takes a value of unity for β equal to unity, as it must.

Such a function may be more appropriate than the low correlation limit as suggested by Kwok[7] and Kareem[8]. As may be seen from *Figure 1*, this latter limit will not be conservative for β less than one.

Vickery *et al.*[9] carried out wind tunnel measurements of the mode shape correction for a square-section build-

ing in two different terrain simulations. Some of these results are also plotted on *Figure 1*, and are seen to agree well with the proposed curve.

The results they obtained for $\beta = 0$ show a considerable amount of scatter and some of these lie outside the limits of 3–4 found here. Apparently, for this uniform mode shape, the mode shape correction is sensitive to the non-uniformities in the spectral density of the sectional force with height. Fortunately, however, this is not a practical mode shape and it is believed that the proposed function is quite adequate in the practical range of $0.5 \leqslant \beta \leqslant 2$.

Also shown in *Figure 1* is the mode shape correction implied in the approximate derivation of gust response factor for along-wind loading by Vickery[10]. This function includes a power law mean wind velocity profile with an exponent α. In *Figure 1*, α has been taken as 0.25 which is the value used for suburban terrain in the Australian Standard[4]. This curve is very close to the proposed curve for β less than unity, but underestimates the experimental data and the proposed curve for β greater than one.

Conclusions

A mode shape correction factor of simple form has been proposed for use in the correction of wind tunnel data obtained for a linear mode shape, and for use in design codes and standards. The function proposed can be used for both along-wind and cross-wind response, and is not dependent on any detailed assumptions on the form of the co-spectrum of the dynamic forces applied by the wind.

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