WIND-INDUCED FATIGUE CYCLE COUNTS – SENSITIVITY TO WIND CLIMATE AND DYNAMIC RESPONSE

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1. Introduction

Fatigue failures produced by turbulent wind forces are relatively common, and a number of authors have predicted damage rates and fatigue lives based on assumed relationships between stress range and cycles to failure (s-N curves), and known wind climates (e.g. Holmes [1], Robertson *et al.* [2], Repetto and Solari [3], Barle *et al.* [4].

An alternative, and earlier, approach, is to specify the number of load repetitions, or exceedences of a certain load, or stress threshold, and not to prescribe the material *s-N* relationship. This approach was proposed by Davenport as early as 1966 [5]. Such an approach is suitable for a loading code or standard, and has in fact been adopted by the Eurocode for Wind Actions [6].

This paper studies the sensitivity of normalized stress exceedence count curves, such as that provided in the Eurocode, to the Weibull wind climate parameters, and to the exponent relating the standard deviation of fluctuating stress to the mean wind speed. Fatigue damage is then evaluated in combination with a standard 3-segment fatigue strength (s-N) relationship used in several codes and standards.

2. Stress cycle count

The Weibull Distribution is commonly used to describe average wind speeds (averaged over 10 minutes to 1 hour) in synoptic wind climates. The distribution which takes the form of Eq. (1).

$$P(>\bar{U}) = \exp\left\{-\left(\frac{\bar{U}}{c}\right)^k\right\} \quad (1)$$

where c is the scale factor and k a shape factor.

The further assumption is made that the standard deviation of the fluctuating stress is related to the mean wind speed by the power law function of Eq. (2).

$$\sigma = A\bar{U}^n \quad (2)$$

The exponent n has a value of 2.0 when a structure reacts quasi-statically to wind loads, but can take a higher value (up to about 2.5) if there is significant resonant dynamic response.

At a site with a wind speed climate described by Eq. (1). the total number of stress cycles with a range exceeding s can be written:

$$N(s) = \frac{k\,\nu_c T}{c^{b+k}}\int_0^\infty \bar{U}^{b+k-1} exp\left[-\left(\frac{\bar{U}}{c}\right)^k - \left(\frac{s^2}{8A^2U^{2n}}\right)\right] d\bar{U} \quad (3)$$

A slightly different version of Eq (3) was derived in Reference [1]. Note that the original version in Ref. [1] was based on stress amplitude (i.e. half range) instead of stress range.

Eq. (3) is based on Rice's level crossing formula [7] which is strictly applicable to narrow-band random processes, and over estimates the cycle count for wide-band processes.

In Eq. (3), the average cycling rate of the stress cycles has been assumed to vary with mean wind speed according to Eq.(4):

$$\nu_o^+ = \nu_1\bar{U}^b = \nu_c\left(\frac{\bar{U}}{c}\right)^b \quad (4)$$

where ν_c is the cycling rate when the mean wind speed is equal to c, and ν_1 is notionally the cycling rate when the mean wind speed is equal to 1 m/s. *b* is an exponent with a value of about 0.5.

T is the total time (in seconds) over which the stress cycles are counted.

Substituting from Eq. (4) into Eq. (3) and changing the variable of integration from $\bar{U}$ to $(\bar{U}/c)$ gives:

$$\frac{N(s)}{k\,\nu_c T} = \int_0^\infty \left(\frac{\bar{U}}{c}\right)^{b+k-1} exp\left[-\left(\frac{\bar{U}}{c}\right)^k - \left(\frac{s^2/(8A^2c^{2n})}{(\frac{\bar{U}}{c})^{2n}}\right)\right] d\left(\frac{\bar{U}}{c}\right) \quad (5)$$

From Eq. (5), it can be shown that the number of stress cycles in the time period T is a function of the following non-dimensional parameters:

$$N(s) = F\,\{b, k, n, \bar{N}, (s/s_{max})\} \quad (6)$$

where $\bar{N}$ is equal to $(\nu_c T)$ [= $(\nu_1 c^b T)$] and is a characteristic number of cycles (or mean crossings) in the time period T.

s_{max} is the expected largest value of stress in the time T, and can be closely approximated by setting $N(s)$ equal to 1 in Eq. (5), and solving it numerically.

Thus, for a fixed set of the four parameters, b, k, n and $\bar{N}$, a single relationship between $N(s)$ and (s/s_{max}) is obtained. Note that Eq. (6) is independent of the parameter A in Eq. (2).

3. Eurocode

Figure B.3 in the Eurocode [6] gives a curve showing the number of times, N_s, that the value ΔS of an effect (e.g. stress) produced by wind action is reached, or exceeded, during a period of 50 years. ΔS is given as a percentage of the maximum load effect, S_k, in the period of 50 years.

The relationship between $\Delta S/S_k$ and N_s in the Eurocode is the following:

$$(\Delta S/S_k) = 1 - 0.174\, log_{10} N_s + 0.007\, (log_{10} N_s)^2 \quad (7)$$

Note that $(\Delta S/S_k)$ is expressed as a fraction in Eq. (7), and as a percentage in [6].

In the following section, the Eurocode function is compared with normalised cycle counts obtained by numerically integrating Eq. (5).

4. Sensitivity and application

By numerically integrating Eq. (5) a sensitivity study was carried out to determine the dependence of the relationship between N(s) and (s/s_{max}) on the various parameters in Eq (6).

For the base case, the following parameters were used:

wind climate:

k = 2.0; c = 6 m/s

dynamic response:

$\nu_1 = 0.1$ Hertz; b = 0.5, n = 2.2

lifetime, T = 50 years

The computed cycle count for this base case is shown in Figure 1, and compared with the Eurocode function (Eq (7).

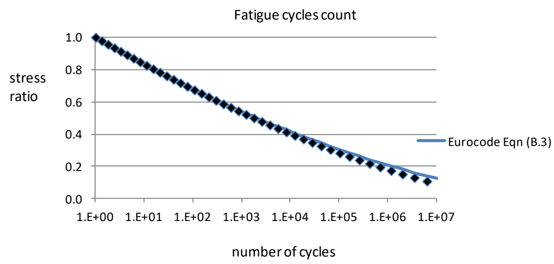

Figure 1. Number of cycles a stress level is exceeded under wind loading

(b = 0.5, c = 6.0 m/s, k = 2.0 and n =2.2; T = 50 years)

It can be seen from Figure 1 that the Eurocode curve (Eq. 7) is generally a good representation of the cycle count for this case - slightly overestimating the cycles at low stress levels.

4.1 Variation with n

Figure 2 shows the variation of the computed crossing rate with n, the exponent in the variation of the standard deviation of stress with mean wind speed (Eq. 2).

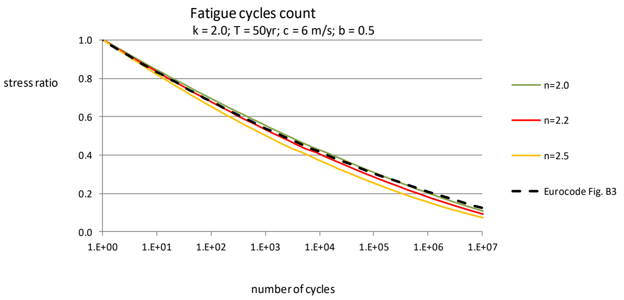

Figure 2. Variation in the number of stress cycles with the exponent, *n*

It can be seen from Figure 2 that the normalised cycle count is relatively insensitive to variation of *n*. There is a slight reduction in the number of stress cycles with increasing *n*.

4.2 Variation with b

Figure 3 shows the variation of the computed crossing rate with *b*, the exponent in the variation of the cycling rate with mean wind speed (Eq. 4).

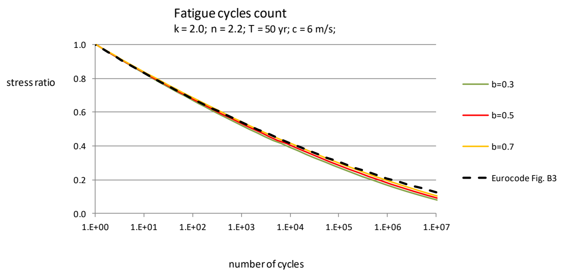

Figure 3. Variation in the number of stress cycles with the exponent, *b*

The stress cycle count is insensitive to the exponent b, with a slight increase in the number of stress cycles at lower stress level as b increases from 0.3 to 0.7.

4.3 Variation with k

Figure 4 shows the variation of the computed crossing rate with *k*, the shape factor in the Weibull distribution (Eq. 1).

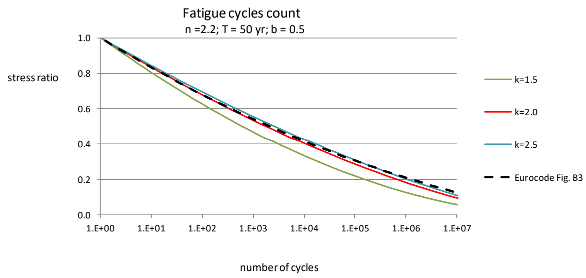

Figure 4. Variation in the number of stress cycles with the Weibull shape factor, *k*

The Eurocode line is a good match for the cycle counts for a shape factor k in the range of 2.0 to 2.5. However, a shape factor of 1.5 produces a reduction in the number of cycle counts, and the Eurocode line is quite conservative for that case.

4.4 Variation with lifetime, T

The variation of the cycle stress count with the reference period *T* (years), which can be assumed to be the required lifetime of the structure, is shown in Figure 5.

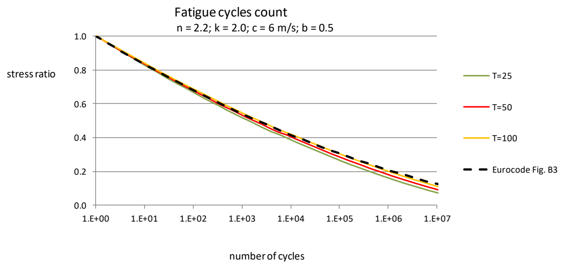

Figure 5. Variation in the number of stress cycles with the lifetime, *T* (years)

The stress cycle count is insensitive to the value of *T* over the range of interest.

4.5 Variation with c

The variation with the Weibull scale factor c (in metres per second) is shown in Figure 6. In fact this is equivalent to a variation in lifetime, *T*, as the only effect of both *c* and *T* is to change the characteristic number of cycles, $\bar{N}$, (see Eq. 6).

Hence, not surprisingly, there is little change of the cycle counts with variation of *c* in Figure 6, just as there was little change in Figure 5 with variation with T.

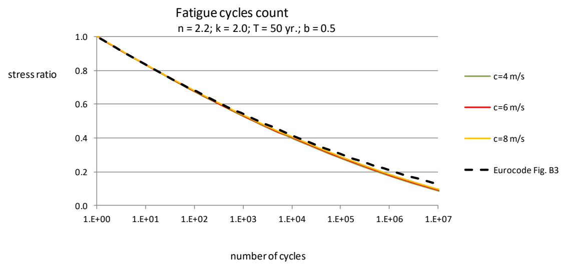

Figure 6. Variation in the number of stress cycles with the Weibull scale factor, c (m/s)

5. Fatigue damage calculation

5.1 Fatigue strength charts

In several codes and standards for steel design (e.g. AS 4100 [8]), the relationship between an allowable stress range, s, of constant amplitude, and number of cycles to failure, N_f, is given by a three-segment function which can be written:

$$N_f\, s^3 = s_D^3 \times 2 \times 10^6, \quad \text{for } N_f \leq 5 \times 10^6 \quad \text{(8a)}$$

$$N_f\, s^5 = s_D^5 \times K, \quad \text{for } 5 \times 10^6 < N_f \leq 10^8 \quad \text{(8b)}$$

$$\text{where } K = (5 \times 0.4^{5/3}) \times 10^6 = 1.0858 \times 10^6$$

$$s = 0.05^{1/5} \times 0.4^{1/3}\, s_D = 0.405\, s_D, \quad \text{for } N_f > 10^8 \quad \text{(8c)}$$

s_D is the 'detail category' and is equal to the allowable stress range for $N_f = 2 \times 10^6$ cycles.

Design charts consist of a number of lines of the form given above with, for various values of detail category, s_D.

5.2 Fatigue damage

Using Miner's Rule, accumulated fatigue damage has been calculated over various stress range sectors determined by the Eurocode function (Eq. 7). Thus, for a sector centred on a stress range, s, the incremental fractional damage is given by:

$$\Delta D = \frac{N}{N_f} \quad (9)$$

where N_f is given by Eq. (8), and N can be determined for a given stress range by Eq. (7).

Then the total damage can be accumulated by summing the contributions from all stress amplitudes:

$$D = \Sigma\, \Delta D = \sum \frac{N}{N_f} \quad (10)$$

Figure 7 shows the calculated damage plotted against the ratio of maximum stress in the defined time period (one crossing), s_{max}, to the detail category, s_D, corresponding to a three segment s-N fatigue strength relationship.

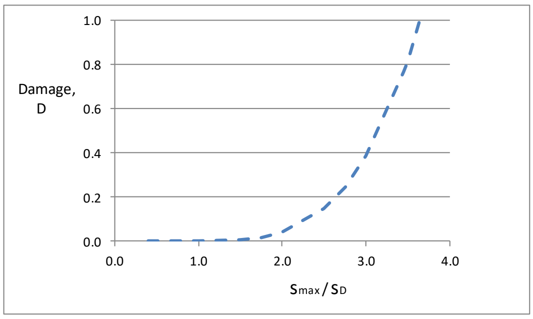

Figure 7. Fatigue damage versus s_{max}/s_D

Damage D equal to 1.0 corresponds to failure. This occurs at a value of (s_{max}/s_D) equal to about 3.65.

Due to the fatigue strength relationship forming a single family of three-segment lines, Figure 7 is a universal graph, applying to all detail categories. Since the stress-cycle count is relatively insensitive to the nominal structure lifetime, or reference period, T applicable to reference periods other than 50 years; however, then s_{max} should be interpreted as the maximum stress in the period of T years.

The damage in Figure 7 can be represented to a good approximation by:

$$\mathrm{D} = 0.0040\left[\left(\frac{s_{max}}{s_D}\right) - 0.405\right]^{4.77} \quad (11)$$

An alternative way to present the same information is in Figure 8, in which the effective fatigue life, calculated from the ratio *T*/*D*, is plotted. *T* is the reference period, taken as 50 years, in this case. Then s_{max} is interpreted as the maximum stress in 50 years.

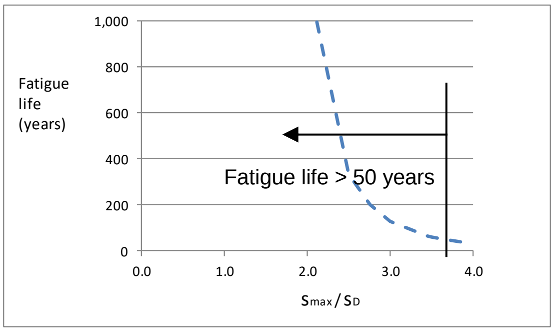

Figure 8. Fatigue life versus s_{max}/s_D, where s_{max} is the maximum stress in 50 years

Clearly, it is desirable to have an estimated fatigue life that is much greater than fifty years. In Figure 8 this occurs when s_{max}/s_D is less than 3.65. Hence a high fatigue life is obtained if the detail category stress, s_D is high, or when the expected maximum stress under wind loading, s_{max}, is low.

The above could be the basis of a simplified design approach to wind-induced fatigue, requiring information on only the maximum stress and the appropriate detail category. However, appropriate partial factors on both the loading and material strength side should be incorporated.

6. Conclusions

Using a theoretical model based on Rice's level crossing formula, and numerical evaluation of the integrals obtained, the sensitivity of normalized stress cycle counts produced by long-term wind action, to parameters related to wind climate (modelled by the Weibull Distribution), and to the dynamic response of the structure, has been determined.

It is found that the cycle counts are insensitive to nearly all the relevant parameters over typical values, although the sensitivity to the Weibull shape factor, *k*, is not insignificant.

The stress cycle relationship given in the Eurocode is found to be a good match to the calculated lines, slightly conservative for low amplitudes – high cycles.

When combined with 3-segment fatigue strength lines, commonly used in steel design, a single relationship is obtained between fatigue damage, and the ratio of the maximum stress in a defined period *T* (years) to the detail category stress characterizing the fatigue strength line. This may lead to a simple design approach to wind-induced fatigue – something that has proved elusive up to now, due to the complexities of both the fluctuating wind loading, and fatigue strength phenomena.

Acknowledgements

The author acknowledges useful discussions of wind-induced fatigue with Dean Genner of Taylor Thomson Whitting, Sydney, Australia, and his suggestion of a simple design approach to wind-induced fatigue.

7. References

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