# Equivalent Static Wind Loading Distributions

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SUMMARY This paper discusses the effective static load distribution approach to wind loading. In this approach, equivalent static load distributions are derived for each of the mean, background (non-resonant fluctuating) and resonant components, and combined. The background loading contribution originates from a formula conceived at the Ruhr-Universitat, Bochum. Although this formula gives a loading contribution which is dependent on the load effect, in many cases simplifications can be made to limit the number of loadings required to be used by designers. The resonant component comprises an inertial loading similar to that used in earthquake engineering.

## 1. INTRODUCTION

Wind loading is an important environmental loading to be considered for the design of structures in most parts of the world. A proper treatment of the wind loading of structures should include consideration of the time-varying nature of the loading. In some respects the dynamic effects are similar to earthquake loading; however there are two significant differences :

(i) in most wind storms and for most structures, the frequencies of the dynamic wind forces are lower than the natural frequencies of the structure, and

(ii) the lack of correlation of the fluctuating forces which results in spatial variations in the forces acting on structures of significant dimensions.

These aspects of wind loading were recognised in the nineteen-sixties by Davenport (1), Vickery (2) and others, and methods were developed for certain large structures such as very tall buildings and large-span bridges, for which the resonant dynamic effects are dominant. However, for the majority of structures, the resonant response is either significant but not dominant, or is negligible. For these structures the sub-resonant fluctuating loading is important and for those with significant tributary areas, the correlation effect, as described in (ii) above, is significant. Another important function for structural design is the influence function for any load effect important in structural design. Wind loading codes or standards, such as Australian Standard AS 1170, Part 2, (3), treat these effects in a rather simplified way.

This paper will treat the problem of wind loading in a general way with particular emphasis on the sub-resonant fluctuating, or 'background' loading and response, and the use of wind-tunnel testing techniques in the most effective way for structural engineers. In particular, the method of equivalent, or effective static wind loading distributions will be introduced. This method enables the instantaneous fluctuating loading distributions associated with the critical load effects for a structure to be identified and combined with the mean wind loading, and with the inertial loads from the resonant response.

## 2. COMPONENTS OF WIND LOADING

Since the pioneer paper of Davenport of 1961 (4), wind speeds, pressures and resulting structural response have generally been treated as stationary random processes in which the time-averaged or mean component is separated from the fluctuating component. Thus

$$x(t) = \bar{x} + x'(t) \qquad (1)$$

where x(t) denotes either a wind velocity component, a pressure (measured with respect to a defined reference static pressure), or a structural response such as bending moment, stress resultant, deflection etc.

$\bar{x}$ is the mean or time-averaged component, and $x'(t)$ is the fluctuating component such that $\overline{x'(t)}$ equals zero.

If x is a response variable, $x'(t)$ should include any resonant dynamic response resulting from excitation of any natural modes of vibration of the structure.

One approach to the calculation of stationary response, including resonance effects, is to expand the complete response as a summation of components associated with each of the natural modes. However, a more efficient approach is to separately compute the mean and background components, as for a quasi-static structure. Thus, the peak response, $\hat{x}$, can be taken to be:

$$\hat{x} = \bar{x} + \sqrt{[x_B^2 + \Sigma(x_{R,j}^2)]} \qquad (2)$$

In most cases in wind loading, only the first mode is significant so that Eq. 2 can be replaced by Eq. 3 to a good approximation.

$$\hat{x} = \bar{x} + \sqrt{[x_B^2 + x_R^2]} \qquad (3)$$

where $x_B$ is the peak background response, and $x_R$ is the peak resonant response computed for the first mode only.

## 3. INFLUENCE COEFFICIENT

A load effect is not the load itself but a parameter resulting from the loading which is required for comparison with design criteria. Examples are: internal forces or moments such as bending moments or shearing forces, stresses, or deflections. The influence line represents the value of a single load effect as a unit (static) load is moved around the structure.

Examples of influence lines are given in Figure 1. Figure 1(a) shows the influence lines for the bending moment and shearing force at a level, S, partway up a tower. These are relatively simple functions; in the case of the shearing force, loads (or wind pressures) above the level s have uniform effect on the shearing force at that level. The influence line for the bending moment varies linearly from unity at the top to zero at the level S; thus wind pressures at the top of the structure have a much larger effect than those lower down, on the bending moment, which, in turn, is closely related to the axial forces in the leg members of the tower. It should be noted that, loads or wind pressures, below the level S have no effect on the shearing force or bending moment at that level.

Figure 1(b) shows the influence line for the bending moment at a point in an arch roof. In this case, the sign of the influence line changes along the arch. Thus wind pressures applied in the same direction at different parts of the roof may have opposite effects on the bending moment, $M_c$.

It is important to take into account these non-uniform influences when considering the structural effects of wind loads even for apparently simple structures, especially for the fluctuating part of the loading.

## 4. MEAN LOADING

The mean wind loading on a structure which does not distort the airflow significantly can be obtained by relating the mean local pressure or force per unit length to the mean wind speed upwind. Thus for the mean along-wind force per unit height acting on a chimney or lattice tower (5) :

$$p_m(z) = [0.5\, \rho\, \bar{u}(z)^2]\, C_d(z)\, w(z) \qquad (4)$$

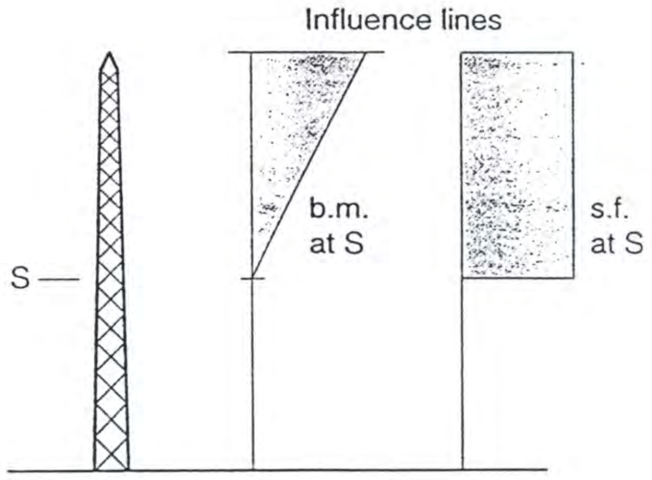
where $\rho$ the density of air

$\bar{u}(z)$ is the mean wind speed at height, z

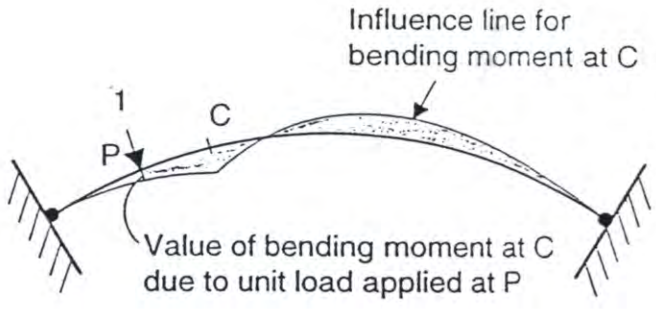
$C_d(z)$ is a drag coefficient for the section at height, z

$w(z)$ is the reference width at the height z

Eq. 4 is already in the form of an equivalent static loading, but the mean value of any load effect (e.g. shearing force, bending moment, deflection) can be obtained by integrating the local load with the influence line over the appropriate height, (6,7).




*(a) Influence lines for bending moment and shearing force in a lattice tower*




*(b) Influence line for bending moment in an arch roof*

*Figure 1.*

In the case of 'solid' structures with at least two dimensions comparable to the size of turbulent eddies in the atmosphere, such as cooling towers and most buildings, Eq. 4 cannot be used, but wind tunnel tests can be used to determine mean pressure coefficients, $\bar{C}_{p,j}$ which can be used with a reference wind speed, $\bar{u}_h$ to determine local mean pressures on the structure :

$$\bar{p}_j = [0.5\, \rho\, \bar{u}_h^2]\, \bar{C}_{p,j} \qquad (5)$$

## 5. BACKGROUND LOADING

The background wind loading is the quasi-static loading produced by fluctuations due to turbulence, but with frequencies too low to excite any resonant response. Until recently the method of defining the equivalent static loading from the background response was not known. Approximations have been made by assuming that the distribution of the background loading was the same as either the mean loading or the resonant loading; clearly both of these assumptions are incorrect. However recent developments have enabled the background component to be included accurately.

Over the duration of a windstorm, because of the incomplete correlations of pressures at various points on a structure, loadings varying both in space and time will be experienced. It is necessary to identify those instantaneous loadings which produce the critical load effects in a structure. The formula which enables this to be done, is the 'Load-Response Correlation' Formula derived by Kasperski and Niemann (8).

This formula gives the 'instantaneous' pressure distribution associated with the expected, or average, maximum or minimum load effect. Thus for the maximum value, $\hat{x}$, of a load effect, x:

$$(p_j)_{\hat{x}} = \bar{p}_j + g_B \cdot \rho_{x,pj} \cdot \sigma_{pj} \qquad (6)$$

where $\bar{p}_j$ and $\sigma_{pj}$ are the mean and r.m.s. pressures at point or panel, j,

$\rho_{x,pj}$ is the correlation coefficient between the fluctuating load effect, and the fluctuating pressure at point j. This can be determined from the correlation coefficients for the fluctuating pressures at all points on the tributary area, and from the influence coefficients.

$g_B$ is the peak factor for the background response which lies in the range 2.5 to 7

The second term on the right-hand side of Eq. 6 represents the background fluctuating load distribution. This term can also be written in the form of a continuous distribution :

$$p_B(z) = g_B\, R(z)\, \sigma_p(z) \qquad (7)$$

where R(z) denotes the correlation coefficient between the fluctuating load at position z on the structure, and the load effect of interest. $\sigma_p(z)$ is the root-mean-square fluctuating load at position z.

In Eq. 6, the correlation coefficient, $\rho_{x,pj}$, can be shown (8,9) to be given by :

$$\rho_{x,pj} = \sum_i \left[\overline{p_i(t)\, p_j(t)\, \beta_i}\right] / (\sigma_{pj}\, \sigma_x) \qquad (8)$$

where $\beta_i$ is the influence coefficient for a pressure applied at position, i.

The r.m.s. structural load effect, $\sigma_x$ is given by :

$$\sigma_x^{\,2} = \sum_i \sum_j \overline{p_i(t)\, p_j(t)}\, \beta_i\, \beta_j \qquad (9)$$

Holmes (9) showed that the application of the above equations can be simplified by using the proper orthogonal decomposition (eigenvalues and eigenvectors) of the covariance matrix of the fluctuating pressures. This approach to background wind loading has been discussed in detail elsewhere, e.g. (10,11), and space does not permit a review of it in this paper.

When the continuous form is used, Eq. 8 and 9 are replaced by an integral form such as the following (5):

$$R(z) = \frac{\int_s^h \overline{p'(z)\, p'(z_1)}\, i(z_1)\, w(z_1)\, dz_1}{\left\{\int_s^h \int_s^h \overline{p'(z_1)\, p'(z_2)}\, i(z_1)\, i(z_2)\, w(z_1)\, w(z_2)\, dz_1\, dz_2\right\}^{1/2} \sqrt{\overline{p'^2}(z)}} \qquad (10)$$

where i(z) now denotes the influence function for the load effect as a function of position, z.

Clearly, since the correlation coefficient, $\rho_{x,pj}$, calculated by Eq. 8, or R(z) calculated by Eq. 10, are dependent on the particular load effect then the background load distribution will also depend on the nature of the load effect.

Figures 2 and 3 give examples of background loading distributions calculated using these methods. Figure 2 shows examples of peak load (mean + background) distributions for a support reaction (dashed) and a bending moment (dotted) in an arch roof, determined from measurements of fluctuating pressures in the boundary-layer wind tunnel at the Ruhr-Universitat, Bochum. These distributions fall within an envelope formed by the maximum and minimum pressure distributions along the arch. It should also be noted that the distribution for the bending moment at C includes a region of positive pressure.

Figure 3 shows the background pressure distribution for the base shearing force and base bending moment on a lattice tower 160 metres high (5), determined by calculation using Eq. 10. The maxima for these distributions occur at around 70 metres height for the base shear and about 120 metres for the base bending moment. An engineering approximation (5) to these distributions is also shown in Figure 3.

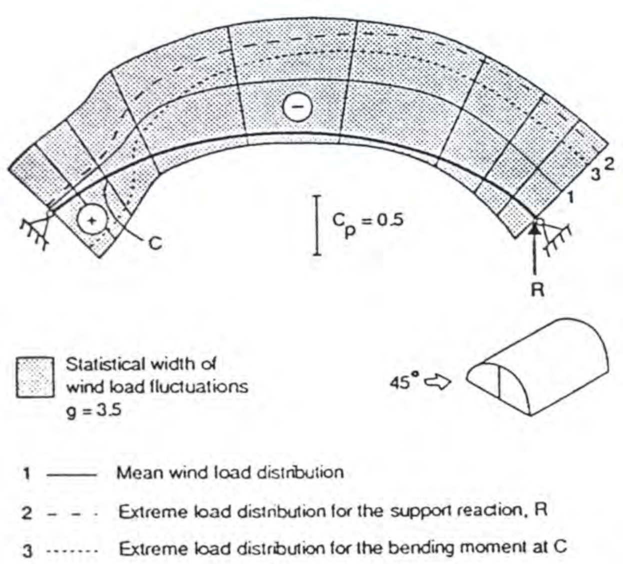



*Figure 2. Mean and background load distributions for an arched roof.*

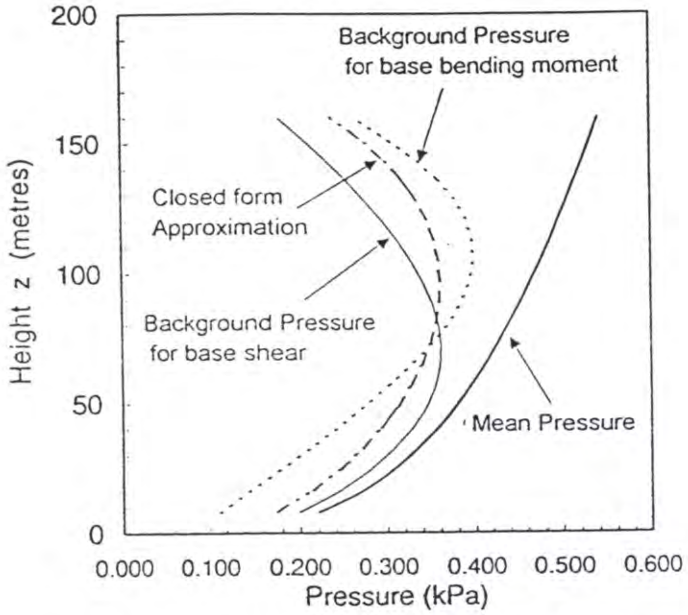



*Figure 3. Mean and background load distributions for a lattice tower.*

## 6. LOAD DISTRIBUTIONS FOR RESONANT RESPONSE

The equivalent load distribution for the resonant response in the first mode can be represented as a distribution of inertial forces over the length of the structure. Thus, an equivalent load distribution for the resonant response, $p_R(z)$ is given by:

$$p_R(z) = g_R \, m(z) \, (2\pi n_1)^2 \sqrt{\overline{a'^2}} \, \mu_1(z) \qquad (11)$$

where $g_R$ is the peak factor for resonant response (3.5 to 4.5)

$m(z)$ is a mass per unit length

$n_1$ is the first mode natural frequency

$\sqrt{\overline{a'^2}}$ is the r.m.s modal coordinate (resonant contribution only)

$\mu_1(z)$ is the mode shape for the first mode of vibration

Determination of the r.m.s. modal coordinate requires knowledge of the spectral density of the exciting forces, the correlation of those forces at the natural frequency (or aerodynamic admittance), and the modal damping and stiffness. However, this is not generally difficult, and, in some cases, can be determined using information given in codes or standards, e.g. (3).

## 7. COMBINED LOAD DISTRIBUTION

The mean, background and resonant loading distributions can be combined in the same way that the load effects themselves were combined (Eq. 3), i.e.

$$p_c(z) = p_m(z) \sqrt{[p_B(z)]^2 + [p_R(z)]^2} \qquad (12)$$

Examples of the combined distribution are given in Figure 4 calculated for a 160-metre lattice tower. When the resonant component is included, the combined loading can exceed the 'peak gust pressure envelope', as is the case in Figure 4, for the bending moment at 120 metres.

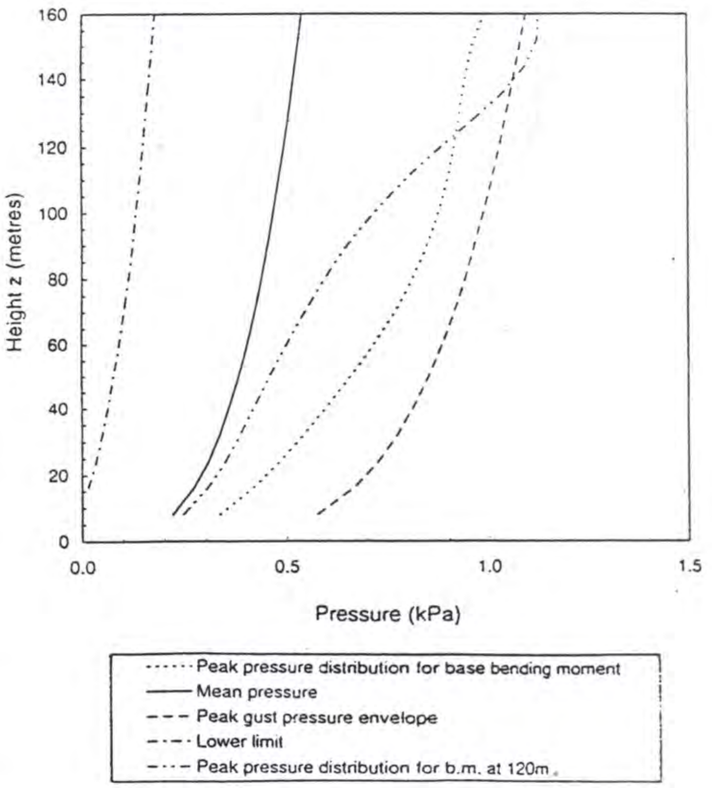



*Figure 4. Combined wind load distributions for a lattice tower*

## 8. CONCLUSIONS

This paper has discussed the effective static load distribution approach to wind loading. In this approach load distributions for the mean, background (non-resonant fluctuating) and resonant loading distributions are combined. The background loading contribution originates from a formula conceived by Kasperski. Although this formula gives a loading contribution which is dependent on the load effect, in many cases simplifications can be made to limit the number of loadings required to be used by designers.

The main advantage of the effective static load distribution approach, over the gust factor approach for example, is that the distributions can be applied to static structural analysis computer programs for use in detail structural design. This approach has been applied to several structures in Australia.

## 9. REFERENCES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