Numerical Solutions for Trajectories of Wind-Driven Compact Objects: Verification and Application

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INTRODUCTION

Strong wind events such as hurricanes can generate wind-borne debris that may cause substantial damage to buildings. As proposed by Wills, Lee and Wyatt [1], wind-borne debris may be classified according to shape into three basic types: "particles" (or compact objects), "sheets" and "rods". This paper, focusing on the first category, describes numerical solutions for the trajectories of spheres and cubes in non-dimensional form and compares the calculated results to tests performed in the wind tunnel. A method of predicting the trajectories of non-ideal compact objects is suggested in the full paper, and some examples of real-world applications are provided.

TRAJECTORIES OF COMPACT OBJECTS

Numerical solutions for the trajectories of spheres have been developed by Holmes [2] for the two cases where the vertical air resistance has been neglected and where it has been included. In the simpler case neglecting the vertical air resistance, a closed-form solution is obtained. When the vertical air resistance is taken into account, no closed-form solution is available, and a numerical approach is required to obtain results. The horizontal velocity of a sphere or cube at impact is shown to be a function of the following variables:

$$u_m = f(\rho_a, U, \rho_m, \ell, C_D, h, g) \tag{1}$$

where ρ_a is the density of air, U is the wind speed, ρ_m is the density of the missile, ℓ is its characteristic length, C_D is the drag coefficient, h is the vertical distance traveled and g is the acceleration of gravity.

The trajectory of a wind-borne missile may terminate when the object strikes a horizontal surface, such as the ground, or a vertical surface, such as the façade of a building. If investigating the potential for wind-borne debris to cause damage to buildings, the latter case is of more interest. In this situation x, the horizontal distance traveled, becomes a known quantity and replaces h in eqn. (1). The horizontal displacement may then be non-dimensionalized as

$$\bar{x} = \frac{xg}{U^2}\,. \tag{2}$$

It is useful to introduce the non-dimensional Tachikawa number, K, [3] where

$$K = \frac{\rho_a U^2}{2\rho_m \ell g}\,. \tag{3}$$

It is now possible to reduce the non-dimensional velocity to

$$\left(\frac{u_m}{U}\right) = f(\bar{x}, K, C_D) \tag{4}$$

which may be calculated and plotted. Figure 1 shows the computed non-dimensional velocity vs. horizontal displacement for varying Tachikawa number for a sphere and a cube. Baker [4] has recently proposed an alternative non-dimensional scheme; this will be discussed in the full paper.

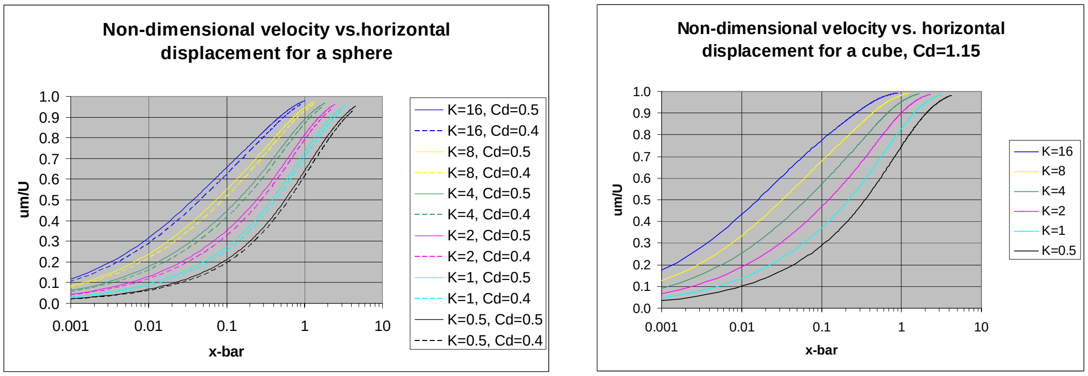

Figure 1. Non-dimensional missile velocity vs. horizontal displacement for a sphere and a cube.

COMPARISON WITH WIND-TUNNEL DATA

Experimental data on the trajectories of wooden spheres and cubes of varying sizes were collected at the wind tunnel at Texas Tech University. Figure 2 shows an example of a comparison of the numerically obtained trajectory with the data from three wind-tunnel runs for a 24.9 mm sphere.

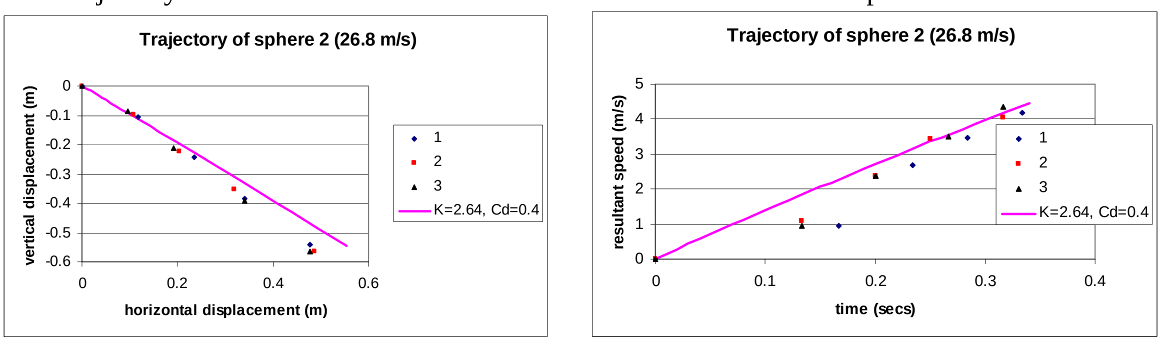

Figure 2. Comparison of numerically calculated trajectory with wind-tunnel data.

CONCLUSIONS

- Numerical calculations have been performed to describe the trajectories of spheres and cubes.
- Comparison of the calculated solutions with experimental data shows very good agreement.
- When non-dimensionalized and plotted, the calculated solutions can be used to predict the trajectories of compact objects at any scale.

ACKNOWLEDGEMENTS

The authors wish to thank Chris Letchford and Ning Lin for the use of wind-tunnel data collected at Texas Tech University and gratefully acknowledge the support of the Louisiana Sea Grant Program.

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